QP CODE: 25024358

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Reg No : .....

Name : .....

M.Sc DEGREE (CSS) EXAMINATION, APRIL 2025

Fourth Semester

M Sc MATHEMATICS

ELECTIVE - ME800403 - COMBINATORICS

2019 ADMISSION ONWARDS

8763C3A2

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. State Addition Principle and Multiplication Principle.
2. In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if the committee must include at least 3 teachers?
3. In how many ways can distribute r identical objects into n distinct boxes , if each box can hold atleast one object.
4. Find a proper edge coloring of 5-clique using blue or red colour in which there is no blue triangle or a red triangle.
5. How will you show that R(p,q) $>$ n for $n \in N$
6. Let A and B be finite sets. Show that $|\bar{A} \bigcap B| = |B| - |A \bigcap B|$
7. Find the number of integers divisible by 7 in {101,102,...,400}
8. Give th formula for E(0)in usual notation
9. Let a_r be the number of partitions of an integer r into parts of sizes 1,2 or 3. Then find the generating function for (a_r).
10. What is mean by recurrence ralation for a sequence (a_n)? Give an example.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Find the number of integer solution to the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{n}$ for $n \in N$

12. Find the number of r-element multi subsets of the multiset $M=\{1.a_1,\infty.a_2,\infty.a_3,\ldots,\infty.a_n\}$

13. Prove that at least 2 people out of 13 assembled in a room must have their birthday in the same month

14. For any integers $p,q \geq 2$ and both R(p-1,q) and R(p,q-1) are even Show that $R(p,q) \leq R(p-1,q)+R(p,q-1)-1$

15. Find the number of non negative integer solutions using GPIE for the equation $x_1+x_2+x_3=11$ where $x_1 \leq 3, x_2 \leq 4, x_3 \leq 6$

16. Show that $\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}(n-1-k)^n=(n-1)!\binom{n}{2}$

17. A)Define ordinary generating function for a sequence (a_r) with an example. Also prove that if $A(x)$ is the generating function for the sequence (a_r), then $(1-x)A(x)$ is the generating function for the sequence (c_r) where $(c_r)=(a_0,a_1-a_0,a_2-a_1,\cdots)$
B) For each $n \in N^*$ let $a_r=(1,1,1,1...)$.Find the closed form of the generating function of sequence (a_r)

18. For each $r \in N^*$, let a_r denote the number of r-digit quaternary sequences in which each of the digits 2 and 3 appears atleast once. Find a_r.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. A)Let $X=\{1,2...,n\}$. Find the number of r- element subset of X that contains no concecutive integers where $0 \leq r \leq n-r+1$
B)A permutation $x_1x_2\ldots x_{2n}$ of $i \in \{1,2,\ldots 2n\}$ where $n \in N$ is said to have the property P if $|x_i-x_{i-1}|=n$ for atleast one $i \in \{1,2,\ldots 2n-1\}$.Show that ,for each n,there are more permutations with property than without

20. Let ABC be an equilateral triangle and E the set of all points contained in the 3 segments AB,BC,CA (including A,B and C). Show that for every partition of E into 2 disjoint subsets, at least one of the 2 subsets contains the vertices of a right angled triangle.

21. Derive the formula for $D(n,r,k)$.Deduce $\lim_{n\to\infty}\frac{D_n}{n!}=\frac{1}{e}$

22. Solve the recurrence relation $a_n-3a_{n-1}=2-2n^2$, given that $a_0=3$.

(2×5=10 weightage)